

## Learning Concepts of Mathematics using GeoGebra at the College Level

**Dr. Madhukar M. Palve,**

Department of Mathematics,

Prof. Ramkrishna More ACS College, Akurdi, Pune,

India – 411044

---

### ABSTRACT:

*This paper presents new trends in technology and learning through Geogebra, which could be especially important for the future development of e-learning for College mathematics. Also, contribution of this paper is presentation of methodological frames on several specifics examples for teaching mathematics at the college level on interactive and creative way.*

**KEYWORDS:** GeoGebra, college mathematics, creative environment.

---

### GeoGebra:

At the moment, GeoGebra (from Geometry and Algebra) is one of the most innovative, open -code math software (GNU General Public License) which can be freely downloaded from [www.geogebra.org](http://www.geogebra.org). GeoGebra works on a wide spectrum of operating system platforms which have Java virtual machine installed on. Markus Hohenwarter created free, open-source dynamic mathematics software GeoGebra, which is used for both teaching and learning mathematics from middle school through college to the University level (see Hohenwarter & Preiner, 2007). GeoGebra offers geometry, algebra and calculus features in a fully connected, compacted and easy-to-use software environment. In other words, this tool extends the concepts of dynamic geometry to the fields of algebra and mathematical analysis. Designed specifically for educational purposes, GeoGebra can help students grasp experimental, problem-oriented and research-oriented learning of mathematics, both in the classroom and at home. Students can simultaneously use a computer algebra system and an interactive geometric system; by doing this, they can increase their cognitive abilities in the best way.

GeoGebra is dynamic geometry software that supports constructions with points, lines and all conic sections. It also

provides typical features for a Computer Algebra System such as finding important points of functions (roots, local extrema and inflection points of functions), direct input of equations and coordinates, finding derivatives and integrals of the entered functions. That is the reason why GeoGebra is a good choice for multiple presentations of mathematical objects.

The basic idea of GeoGebra's interface is to provide two presentations of each mathematical object in its algebra and graphics windows. If you change an object in one of these windows, its presentation in the other one will be immediately updated. Computer algebra systems (such as Mathematica, Maple) and dynamic geometry software (such as Geometer's Sketchpad, Cabri Geometry, and so on, e.g.) are powerful technological tools for teaching mathematics. Numerous research results suggest that these software packages can be used to encourage discovery, experimentation and visualization in traditional teaching of mathematics. However, researches suggest that, for the majority of teachers, the main problem is how to provide the technology necessary for the successful integration of technology into teaching (Ruthven & Hennessy, 2004).

So, the suggested solution for applying technology in the college math teaching and learning is the software packet GeoGebra..

**The advantages of using GeoGebra are**

- 1) In comparison to a graph calculator, GeoGebra is more user-friendly. GeoGebra offers easy-to-use interface, multilingual menus, commands and help.
- 2) GeoGebra encourages students' projects in mathematics, multiple presentations and experimental and guided discovery learning.
- 3) Students can personalize their own creations through the adaptation of interface (e.g. font size, language, quality of graphics, color, coordinates, line thickness, line style and other features).
- 4) GeoGebra was created to help students gain a better understanding of mathematics. Students can manipulate variables easily by simply dragging "free" objects around the plane of drawing, or by using sliders. Students can generate changes using a technique of manipulating free objects, and then they can learn how the dependent objects will be affected. In this way, students have the opportunity to solve problems by investigating mathematical relations dynamically.
- 5) Cooperative learning is the right context for a mathematics course. Lecturing should be replaced by a task-oriented interactive classroom. The primary role of teaching is not to lecture, explain, or otherwise attempt to "transfer" mathematical knowledge, but to create situations for students that will foster their making the necessary mental constructions. In

that sense, GeoGebra provides a good opportunity for cooperative learning, i.e. cooperative

problem solving in small groups, or whole class interactive teaching, or individual/group

student presentations.

- 6) The algebra input allows the user to generate new objects or to modify those already existing, by the command line. The worksheet files can easily be published as Web pages.
- 7) GeoGebra stimulates teachers to use and assess technology in: visualization of mathematics; investigations in mathematics; interactive mathematics classes on site or at a distance; mathematics and its applications, etc.

**The deficiencies of using GeoGebra are:**

- Students without previous programming experience will hardly enter algebraic commands in the input box. Although the basic commands are not difficult to learn, students may feel embarrassed or quite at a loss of what to do.
- Some methodological approaches (e.g. Independent exploring and experimenting) cannot be appropriate for many students.
- In a technical sense, GeoGebra does not have an in-built support for animation. So, including the modules for animating in GeoGebra should become an important technical element for future versions.
- Future extensions of the software GeoGebra will surely include moresymbolic features of computer algebra systems which will further increase possible complex applications in the mathematical analyses, and 3D extensions.

- Limited research on the impact of GeoGebra on teaching and learning of mathematics.

GeoGebra has been rapidly gaining popularity among teachers and researchers around the world, because it is easy-to-use dynamic mathematics software that combines many aspects of different mathematical packages. In addition, because of its open-source nature, an extensive user community has developed around it.

### **Math Visualization and Exploration by Using GeoGebra**

The visualization that is possible with today's dynamic software enables the student to see and explore mathematical relations and concepts that were difficult to "show" in past prior to technology. The most meta-analysis of research in the area of instructional technology in education show that students who use technology in their learning had positive gains in learning outcomes over students who learned without technology

Although my teaching and learning methods are still traditional (textbook, lecture notes, lectures, assignments, final exam) my plan is to try to make some changes in the way of work in order to

make my teaching courses more effective to improve the learning outcomes of my students. The goal is to use GeoGebra to provide an environment for active exploration of mathematical structures through multiple representations, or to show students some aspects of the mathematics that are not possible with pen and paper.

### **Interactive Examples in Teaching and Learning Linear Algebra with**

#### **GeoGebra:**

Using examples and questions given below, the teacher will guide students to fully understanding of the meaning and nature of solutions to linear systems of equations. These examples illustrate low-order systems to assist students in personalizing important concepts about systems of linear equations by "discovery method". The teacher should define the terms "consistent, inconsistent, homogeneous", "one solution, no solution, infinite solutions", and precise definitions of "linear equation, linear system, nonlinear system," and "solution". Also, the teacher should strongly encourage geometric visualizations through the lesson plan (Dikovic, 2007).

Example 1. Consider families of equations which take the form

$$ax + by = c. \dots \quad (1)$$

For different values of  $a, b, c$ , the graphs of these equations are lines in a two dimensional coordinate system. Let the students choose any real numbers  $a_1, b_1, c_1, a_2, b_2, c_2$  to define a system of two equations in two unknowns:

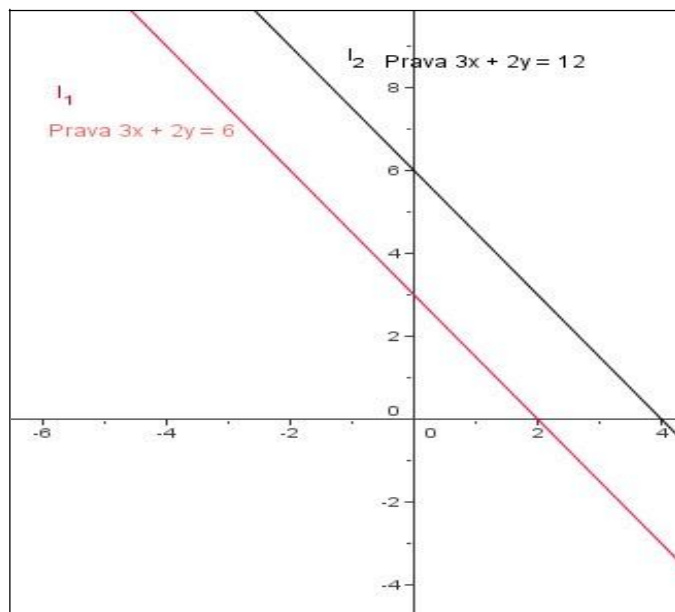
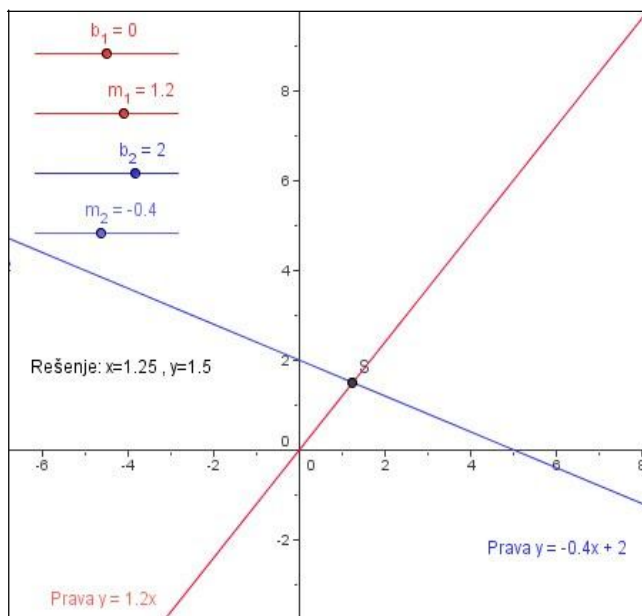
$$\begin{aligned} a_1 x + b_1 y &= c_1, \\ a_2 x + b_2 y &= c_2. \end{aligned} \quad (2)$$

These graphs can easily be generated using GeoGebra. GeoGebra allows modifying these equations dynamically by moving the points or the sliders. In this way, students

are able to investigate all possible situations including the interesting special cases of no solution and infinitely many solutions

Q1. Can you change the numbers of the second equation so that the resulting system has

- No solution? (see figure 2)
- Exactly one solution? (see figure 1)
- Infinitely many solutions?
- If you do get one (infinite) solution for your final answer, is this system consistent or inconsistent? Would the equations be dependent or independent?



Q2. Student can set  $a_1, b_1, a_2, b_2$  to values such that

$$a_1 \cdot b_2 - a_2 \cdot b_1 = 0 \quad \text{.....} \quad (3)$$

and consider the next questions: How are the two lines positioned with respect to each other? How many solutions the system has? What is the relationship between the slopes of the two lines?

Q3. Student can set all 6 constants to values such that

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \quad (4)$$

and explain how many solutions the system has in that case.

Q4. Ask the students to randomly select numbers of a third equation and to graph it along with the first two. Normally such a random selection should generate an inconsistent system with a large probability, whose lines bound a triangular region. Does

this new system have a solution and whether they have a solution or not?

Also, GeoGebra brings to students and teachers the opportunity to individualize learning - to generate illustrative examples, to follow interesting topics to the desired

depth, to choose their own problems and appropriate tools for solving them. The students can concentrate on ideas instead of trying to get the arithmetic right in the solution of the some linear system.

### Conclusion:

In this paper, I highlighted some opportunities and examples on how GeoGebra can be used in classrooms to explore some basic concepts in linear algebra and calculus. It was showed that GeoGebra has many possibilities to help students to get an intuitive feeling and to

Additionally in examples of solving linear system, GeoGebra can stimulate the students' geometric intuition through interesting visualizations in 2D.

visualize adequate math process. The use of this software's tools allows students to explore a wider range of function types, and provides students to make the connections between symbolic and visual representations.

### References:

- van Joolingen, Wouter (1999): *Cognitive tools for discovery learning. International Journal of Artificial Intelligence in Education*, 10, p. 385-397.
- [www.bsrim.org.uk@theauthor-49](http://www.bsrim.org.uk@theauthor-49)
- NCTM (2000). *Principles and Standards for School Mathematics*
- <http://standards.nctm.org>
- Herceg Dj., Herceg D.(2007), *Numerics with GeoGebra in high school, First Central- and Eastern European Conference on Computer Algebra- and Dynamic Geometry Systems in Mathematics Education*, University of Pécs, Pollack Mihály Faculty of Engineering, Hungary.
- Dikovich Lj.(2007), „An Interactive Learning and Teaching of Linear Algebra by Web Technologies: Some Examples“, *Journal the Teaching of mathematics*, Publisher: The Mathematical Society of Serbia, Beograd, ISSN: 1451-4966, Issue: X\_2, Pages: 109 – 116.
- Wurnig O. (2008), *Some Problem Solving examples of Multiple solutions using cas and dgs, Proceedings of the Discussing Group 9 : Promoting Creativity for All Students in Mathematics Education, The 11th International Congress on Mathematical Education, Monterrey, Mexico..*
- <http://www.geogebra.org>
- <http://www.learning-theories.com/discovery-learning-bruner.html>